FALSE STATEMENT IN THE *SOPHIST* AND THEAETETUS' MATHEMATICS

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THE PURPOSE OF THIS PAPER is to call attention to a parallel between Plato's account of false statement in the *Sophist* and Theaetetus' study of incommensurables, substantially preserved for us in Euclid's *Elements*, Book 10.

The initial attempt to explain the nature of the sophist produced a multiple definition (218b–231c, summed up at 231de), indicating obscurity as to what he really is. On closer inspection, he seemed to be one who professes to teach how to argue persuasively on any subject without knowing it oneself (for no one can know everything: 233a). Thus he claims to know how to make oneself appear to know what one does not, by stating what appears to be the truth about it, but is not. And if so, then he is a kind of illusionist, a maker of deceiving images in words, mere semblances (*phantasmata*) rather than true likenesses (*eikones*), leading men to mistake these false statements for true ones, and to mistake him, who knows nothing in particular, for the wisest of men on all subjects (232b–236c). But this definition can be sustained only if it can be shown that there really is such a thing as appearing, or being stated or thought to be, but not being. One must refute Parmenides' argument that not-being is "unthinkable, inexpressible, unspeakable, irrational" (236c–241c). Theaetetus and the Eleatic Stranger reason it out that, since being any specifiable subject implies not being any other, different subject (256e, *contra* 237be), and since *not being* is being other than or different from some specifiable other, different thing (258b, *contra* 238–239b), there plainly is such a thing as not-being, viz., being other than, or differing from, something or other (254d–259d).[1] It remains to show that there is also such a thing as not-being in discourse, thought, or appearance, viz., being other than, or different from, something or other that the subject is falsely stated, thought, or made to appear to be (260b–261c).

The groundwork for this demonstration is laid down in a theory of statement (261d–262), the gist of which is as follows. We use two kinds of sign to indicate what there is: verbs to signify actions and names to

[1]Compare 237be and 238–239b with the fifth and sixth hypotheses of the second part of the *Parmenides*, 160b–163b and 163b–164b, which argue, respectively, that a subject of statement or thought must be something or other, and that what is nothing whatever is no subject at all. In short, nothing simply is, or simply is not: both being and not-being are "incomplete" predicates, like being large, being like, and being one. See G. E. L. Owen, "Plato on Not-Being," in G. Vlastos (ed.), *Plato: Critical Essays* (Garden City 1970) 1.223–267.

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signify agents (261e4–262a7). A statement, however, is not just a list of actions or agents. It does not consist simply of a string of verbs or of names; just as not all forms blend, so not all signs fit together so as to form a statement. Rather, a statement combines, or interlaces, at least one name with at least one verb, whereby it states one thing and does not merely list several. It thus states an action done or not done, or what (something) is or is not (262c3). In this way, it gives information about what is or becomes or has or will become (262d–263). Thus a statement states, about a subject that is something, not nothing (262e5–6), some one thing that is true or false (262d4, 262e8, 263b2–3).

The Stranger proceeds now to define truth and falsity. After giving the examples "Theaetetus sits" and "Theaetetus flies" he reasons as follows:

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ΞΕ. Λέγει δὲ αὐτῶν ὁ μὲν ἀληθὴς τὰ ὄντα ὡς ἔστιν περὶ σοῦ.
ΘΕΑΙ. Τί μήν;
ΞΕ. Ὁ δὲ δὴ ψευδὴς ἕτερα τῶν ὄντων.
ΘΕΑΙ. Ναί.
ΞΕ. Τὰ μὴ ὄντ' ἄρα ὡς ὄντα λέγει.
ΘΕΑΙ. Σχεδόν.
ΞΕ. Ὄντων δέ γε ὄντα ἕτερα περὶ σοῦ. πολλὰ μὲν γὰρ ἔφαμεν ὄντα περὶ ἕκαστον εἶναί που, πολλὰ δὲ οὐκ ὄντα.
ΘΕΑΙ. Κομιδῇ μὲν οὖν.
ΞΕ. Ὃν ὕστερον δὴ λόγον εἴρηκα περὶ σοῦ, πρῶτον μέν, ἐξ ὧν **c** ὡρισάμεθα τί ποτ' ἔστι λόγος, ἀναγκαιότατον αὐτὸν ἕνα τῶν βραχυτάτων εἶναι.
ΘΕΑΙ. Νυνδὴ γοῦν ταύτῃ συνωμολογήσαμεν.
ΞΕ. Ἔπειτα δέ γε τινός.
ΘΕΑΙ. Οὕτως.
ΞΕ. Εἰ δὲ μὴ ἔστιν σός, οὐκ ἄλλου γε οὐδενός.
ΘΕΑΙ. Πῶς γάρ;
ΞΕ. Μηδενὸς ⟨δέ⟩ γε ὢν οὐδ' ἂν λόγος εἴη τὸ παράπαν· ἀπεφήναμεν γὰρ ὅτι τῶν ἀδυνάτων ἦν λόγον ὄντα μηδενὸς εἶναι λόγον.
ΘΕΑΙ. Ὀρθότατα.
ΞΕ. Περὶ δὴ σοῦ λεγόμενα, ⟨λεγόμενα⟩ μέντοι θάτερα ὡς τὰ αὐτὰ **d** καὶ μὴ ὄντα ὡς ὄντα, παντάπασιν [ὡς] ἔοικεν ἡ τοιαύτη σύνθεσις ἔκ τε ῥημάτων γιγνομένη καὶ ὀνομάτων ὄντως τε καὶ ἀληθῶς γίγνεσθαι λόγος ψευδής.

STR: The true one states things that are, as they are, about you. THT: Certainly. STR: Whereas the false one states things that are different from those that are [about you]. THT: Yes. STR: Or, it states things that are not, as things that are. THT: Roughly. STR: Things that certainly are, but things other than those that are about you. For we said concerning each thing there are many things that in a way are [254b–257a] and also many that are not [257c–258].[2] THT: Precisely. STR: That last statement ["Theaetetus flies"] which we said about you, in the first place, from the way we defined the essence

[2]Each thing is identical with itself and non-identical with every other thing; it is, by participation, indefinitely many things, and again other than indefinitely many. We read

of a statement, must certainly be one of the very shortest. THT: We did agree on that just now. STR: And secondly it was about something? THT: Yes. STR: If not about you, about no other thing? THT: How do you mean? STR: Were it about nothing, it would not even be a statement at all; for we denied that a statement about impossibilities, being about nothing, was a statement. THT: Entirely right. STR: So, when things are said about you, but things other [than those that are about you] as the same, or, things that are not as things that are, it seems very likely that such a combination formed of verbs and names really and truly is false statement.

The Stranger and Theaetetus then extend the argument to thought and appearance (263d–264b).

The Stranger emphasizes a point he had made when formulating the examples: the statement must have a subject that is something, not nothing, viz., "the Theaetetus to whom I am now talking." Moreover, the property predicated is likewise something, not nothing (263b11). The false statement is false in that this property that it states Theaetetus to have is not one that he does have; or, it is his property in statement only, not in being. In general, a false statement states about a subject (that is something) things that are (are something), but that are not with respect to it, i.e., it states things that the subject is not, as if they were things that it is.

Let us apply this definition to the examples given in the text. The true and false cases are as follows:

(1) "Theaetetus sits" is true, in that:

As "Theaetetus sits" *says*, of Theaetetus, that he sits, so he *does* sit.

(2) "Theaetetus flies" is false, in that:

Whereas "Theaetetus flies" *says*, of Theaetetus, that he flies, yet he *does not* fly.

Or, in Plato's more compressed language, the first says of Theaetetus things that are the same as things that he is. The second says of him things that are other than things that he is.

We may put a generalized version of (1) and (2) this way. If "X is F" is a statement of the simplest sort indicated by the Stranger, "X" representing any subject and "F" any property,

(3) "X is F" is true, provided that:

As "X is F" says, of X, that it is F, so it is F.

(4) "X is F" is false, provided that:

Whereas "X is F" says, of X, that it is F, yet it is not (is other than) F.

It is illuminating to read (3) and (4) as defining truth and falsehood by way of proportion and disproportion, respectively, as follows:

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the passage at 257c–258 in accord with the interpretation argued by Edward N. Lee in a paper "Negation and Non-Being in the *Sophist*," forthcoming in *Philosophical Review*.

(5) "X is F" is true =df.
As "X is F" relates X to F, so X is related to F.
(6) "X is F" is false =df.
As "X is F" relates X to F, *not* so is X related to F.

The effect of these definitions is to treat the subject and the property it is said to have as four terms arranged into two pairs, one of which relates the subject and property in statement, while the other relates them in the world. For by lacing a verb with a noun, a statement predicates a property of a subject, and thereby states them to have a relation which may be either the same as or other than the relation which they actually have. Sameness amounts to truth, otherness to falsity.

The application of the notion of proportion[3] to the analysis of truth and falsehood presupposes that where A:B::C:D, there must be homogeneity between A and B, and again between C and D, but not necessarily between A and C (Cf. *Elements* 5, Deff. 3,5).[4] The content of a statement of proportion is that the relation or ratio of the pairs of terms, whether taken from the same or from different fields of being, is nevertheless the same. Where the two pairs are in fact heterogeneous, as they often are, the statement of proportion will be elliptical, requiring completion as follows: A is to B (in respect of J) as C is to D (in respect of K). Consider the examples:

(*a*) Of triangles under the same height, one is to another (in area) as their bases are (in length).
(*b*) Evening is to morning (in respect of day) as old age is to youth (in respect of life).
(*c*) Simmias is next to Cebes (in a picture) as he is next to him (in reality).
(*d*) Vowels are to syllables (in forming minimum pronounceables) as verbs are to sentences (in forming minimum statables).
(*e*) (*i*) Sophistry is to politics (in maintaining the soul) as cosmetics is to gymnastics (in maintaining the body).

[3]Plato frequently exploits the resources of this device: see, e.g., *Soph.* 262de (cf. 252e–253a, 261de); *Tht.* 148d, 185de (a disproportion), 202b; *Ti.* 29bc, 31b–32c, 87c–d; *Grg.* 464b–465 (esp. 465bc), 520ab; *Resp.* 441a, cd, and 533e–534a, 508bc, 509a, 509d–511 (esp. 510a and 511de). These passages show conclusively that Plato uses λόγος with an awareness of its technical geometrical sense. It is not improper, then, to anticipate some borrowing from mathematics in our passage: after all, Plato's spokesman is talking with an expert in this very branch of mathematics, theory of proportion. See below, notes 7, 8 and 14.

[4]Ian Mueller, "'Homogeneity' in *Elements*, Book V," *Archive for History of Exact Science* 5 (1971), investigates this restriction in Euclid and concludes that it had not been imposed by Eudoxus in his earlier version of Book 5. Eudoxus' concept "magnitude" already incorporated all the needed "homogeneity."

(*ii*) Rhetoric is to justice (in restoring the soul) as cookery is to medicine (in restoring the body).

What is the identical relation asserted in each of these examples? In (*a*) it is a measuring, or "part(s) of" relation;[5] in (*b*) it is being near the end and beginning respectively of a continuous stretch of time; in (*c*) it is contiguity; in (*d*) it is being a necessary constituent in a unit of expression; in (*e*) it is impersonation. Applying this to the analysis of true and false statement: "Theaetetus sits" is true, in that Theaetetus is related to sitting, in statement (i.e., in the statement that he is sitting), as these are related in being (see [5] above). The relation that holds the same is "having part in," in Plato's technical vocabulary. Thus Theaetetus is said to be sitting, and he is sitting, and so what is said is true to what is.[6]

Plato's interest in explaining falsehood as a failure of proportion would have been motivated at least partly by the fact that it was the mathematician Theaetetus who had given a parallel explanation of the incommensurability of sizes. There is good reason to believe that it was Theaetetus who did the fundamental research behind at least the definitions and first nine theorems of Book 10 of the *Elements*. The four main sources from which we can reconstruct the general outlines of Theaetetus' work, Aristotle, Eudemus of Rhodes, Pappus, and an anonymous scholiast to *Elements* 10, have been reviewed, together with the important work on this material by Oskar Becker, in a recent article,[7] and need not be gone over in detail here. The net result of this review is to credit to Theaetetus at least the fundamental research behind, if not the exact wording of, the theory of proportion whose most general theorems survive at the beginning of *Elements* 10. The evidence is still stronger which shows that Theaetetus was an associate of Plato's at the Academy, where he joined others in researches in the mathematics of incommen-

[5]"Being a part" must be understood one way for commensurable cases, another way for incommensurable ones. Cf. Arist. *Metaph.* Δ, 25: "part of" can relate A and B either restrictedly or unrestrictedly (1023b12–17). In the restricted meaning the lesser, A, must divide B without remainder; in the other meaning, some unit (e.g., the lesser) must divide both A and B without remainder. The restricted definition of "part" appears in both Book 7 (the earlier Pythagorean theory of proportion) and Book 5 (the generalized one). But in 7 the restriction carries over to proportion: the four terms must all be measurable-part(s), the lessers of greaters. In Book 5 none of the terms need be so with any other.

[6]Thus the theory of truth and falsehood is equivalent to the familiar correspondence theory. As has often been noted, this does not formulate a *criterion*, but an *analysis*, of truth. It does not help one decide which statements are true, but only shows in a general way what is being said of any statement when one says that it is true.

[7]Malcolm Brown, "*Theaetetus:* Knowledge as continued learning," *Journal of the History of Philosophy* 7 (1969) 362–363.

[8]On the first point, his mathematical researches at the Academy, the famous "catalogue of geometers" preserved from Eudemus (see Morrow's translation of Proclus on

surables, foundations of mathematics, and in philosophy.[8] In Book 10 we find the proof that the sufficient and necessary condition of the commensurability of sizes is that they be in the same ratio as a pair of whole numbers (props. 5, 6), and of their incommensurability, that they not be in such ratio (props. 7, 8):

(7) A and B are commensurable, provided that:
As A is to B, so is one whole number to another.

(8) A and B are incommensurable, provided that:
As A is to B, *not* so is one whole number to another.

Propositions 5 through 8 are summarized in proposition 9 and applied to the case of pure quadratic incommensurability. (This is the sort of incommensurability referred to at *Tht.* 147–148.)[9] The simplest and most familiar instance of this is given by the side and diagonal of the square. One part of proposition 9 says: "squares which have not to one another the ratio (λόγον μὴ ἔχοντα, ὅν: *Euc.* 3. 24 Heiberg) of a square number to a square number will not have their sides commensurable in length either." The square on the diagonal has the relation to the square on the side that 2 has to 1, by the Pythagorean Theorem. But it is easily shown that no pair of square numbers exists that stand in the relation 2 to 1. Consequently the diagonal and side are incommensurable. That is to say, "The diagonal of a square is commensurable with its side" is false,[10] in that, as it says that the diagonal and side are related, *not* so are any square numbers related: there are things which it states these sizes to be, and which they are not (viz., in the ratio of a square number to another square number). (Compare [2], [4], and [6] above.)

Let us now summarize by drawing out some parallels between Plato's explanation of false statement and Theaetetus' handling of incommensurables. First, the explicit form of assertions of incommensurability and of falsehood is "as . . . so . . ." and "whereas . . . yet . . .," or "as . . . *not*

Euclid 1. 54–56 and footnote 37) is our best source. On the point about his ability at philosophy, cf. *Tht.* 155d, where Socrates pays Theaetetus the unusual compliment about his philosophical temperament.

[9]In this passage Plato credits Theaetetus, in his youth, with investigating only two kinds of surds, pure quadratics and pure cubics (e.g., $\sqrt{3}$ and $\sqrt[3]{3}$). Book 10 has many more, including medials and binomials (e.g., $\sqrt[4]{6}$ and $1+\sqrt{3}$). Plato may be acquainted also with such compound irrationals (cf. *Hp. Ma.* 303bc).

[10]Proclus, alluding to *Resp.* 546 c (see Morrow, translation of Proclus on Euclid 1. 49 and note 20), proposes to express the ratio of a square's diagonal and side by means of "approximations," clearly the same ones familiar to Plato from the ancient Pythagorean method of "side- and diagonal-numbers." Proclus says: "When, in geometry, we have found a square double a given square but do not have it in numbers, we say that a square number is the double of another square number when it is short by one, like the square of seven, which is one less than the double of the square of five." To say that "$7^2/5^2 = 2/1$" is only an "approximation" because it is "short by one" is as much to say that the statement is *false* as to say that the thing stated to be so is *not*.

so . . . ." (Compare [3] and [4] with [7] and [8] above.) Second, the fillers in these blanks are *logoi* in the sense of two terms bound in a relation, this pair being put together according to a definition of *logos*[11] (262d; cf. *Elements*, Book 5, Def. 3). Third, the content of such assertions is the sameness (or difference) of the relation within each pair (some manuscripts of *Elements* 5 have the gloss on its definitions of proportion and disproportion: ταυτότης τῶν λόγων and ἑτερότης τῶν λόγων). Fourth, the negation involved in denying proportion in each case can occur as prefix of the entire four-term arrangement, or of the sameness between the pairs, or of a third and a fourth term in the specified relation or of a fourth term only. Thus the following are equivalent: "It is not the case that A is to B as any C is to any D," "A is to B not as any C is to any D," "There are no C and D such that A is to B as C is to D," and "There is no D such that, A is to B as any C is to it." The final point relates to this last, negative-existential formulation: it is this one which especially excites perplexity, both in mathematics and in philosophy. An ancient scholion to Book 10 expresses the difficulty about incommensurability of side and diagonal of a square as if denying the existence of the *ratio* 2:1 among square numbers amounted to denying the existence of a fourth *term* in a proportion.[12] An exactly parallel account is given by Proclus, commenting on Plato's allusion to incommensurability at *Resp.* 546c: "there is not [οὐ γάρ ἐστιν] a square number double of a square number."[13] The difficulty can also be put as follows: since the relation of a square's side to diagonal is as $1:\sqrt{2}$, and since no whole numbers are so related, how can there *be* such a mathematical entity as $\sqrt{2}$?[14] Likewise, a

[11]Book 5, Definition 3 attempts a definition of "logos," but, according to Heath, gets a metaphysical, not a mathematical, result. The point raised by Heath is that the definition seems only to distinguish slightly the meaning here from that in common life. A number of passages from Plato and Aristotle show that this two-term form was not proprietary to the mathematicians: *Grg.* 464b–465; *Poet.* 21. 11–14, and *N. E.* 5. 3–5. See further B. Einarson, "On Certain Mathematical Terms in Aristotle's Logic," *AJP* 57 (1936) 33–54, 151–172.

[12]The scholion says: "Square numbers having the ratio 2:1 to one another could never be (οὐδέποτ' ἂν εἶεν). For none of the squares will be found to have the double ratio, not 4, 9, 16, 25, 36, or any of the rest of the squares in sequence. For putting these together there will be found not a one (οὐδεὶς) having the double ratio to any other whatever (πρὸς ἄλλον ὁντιναοῦν) (*Euclidis Elementa* 5. 423.16–424.1; Book 10, Scholion 8). Cf. Proclus, cited above, note 10. Each square number *is*, but none *is* in just the relation to any other that 2 is to 1. The not-being turns out to be not of any fourth term in itself, but as fourth term in *that* relation to some third.

[13]Proclus, *In Rempublicam Commentarii* ed. G. Kroll (Leipzig 1901) 2. 27.2–3.

[14]Or, for that matter, how can there be any ratio (λόγος) of the unit to such a thing? That ratio would seem to be inexpressible (ἄλογον). Cf. *Elements* 10, Deff. 3 and 4; and *Soph.* 238–239b, where it is said three times that not-being is ἄρρητον and ἀφθεγκτον and ἄλογον. If one had to pick two Greek terms to recall the subject of the mathematical research of Theaetetus, the ἄρρητον and the ἄλογον would be excellent candidates. In

property that Theaetetus does not have or is falsely stated to have, is, apparently, one that does not exist; for as regards Theaetetus, flying is simply not there, hence not there to be stated, even falsely of him: there is no such property at all. But (and this is the crucial point) explicit formulation in the language of proportion has in both cases the advantage that the apparent not-being of a fourth *term* is revealed as the not-being simply of the sameness (i.e., as the difference) of two *relations* (see above, [5], [6]). This not-being is not merely compatible with, but implies, the being (the being something) of all four terms, the relation of whose pairs (which pairs also are something) *is not* the same.

The parallel seems to fail in one respect. Whereas Theaetetus in *Elements* 10 explains incommensurability as otherness of relation among two *different* pairs of terms, the account we here attribute to Plato's Stranger explains falsehood as involving the *same* pair of terms, related otherwise in being than in statement. This apparent dissimilarity, however, is mitigated by the following reflection. Our geometrical example (*a*), expressed in canonical Euclidean form as a proportion among four terms, can be trivially reformulated as holding between just two, namely,

(*a'*) Of triangles under the same height, one is to another (in area) as the one is to the other (in length of base).

Here the very same terms are said to stand in just the same relation, viz., sameness of measure, applied now in respect of area, and again in respect of base length. In the same way, to say that "Theaetetus sits" is true is to say that Theaetetus and sitting are related in the statement in just the way that they (these very same two) are related in being. Example (*c*) reinforces the idea that an identical pair of things may, in a particular case, be of the form: A is to B (in respect of J) as A is to B (in respect of K, where J $\neq$ K): Simmias is next to Cebes pictorially as he is next to him really. We will return to this point below. Meanwhile, however, we can speak to a second apparent dissimilarity, namely that whereas (8) prescribes that A and B are incommensurable when not in the ratio of *any* whole number to *any* other, (6) prescribes that X is falsely said to be F when *that specific* X is other than that specific F. In other words, (8) seems to have a generality absent from (6). However, both (8) and (6) are meant to hold good for *any* suitably chosen pairs (A, B) and (X, F) respectively. Thus while (8) requires quantification of its third and fourth terms also, this has already been done by implication in (6), inasmuch as its third and fourth terms are the same as its first and second terms. Here we no longer have an ordinary four-term proportion, but four

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the same passage Plato's Stranger suggests that the young Theaetetus may have his own ὀρθολογία about not-being (239b4). And he even strains the language to make this point: ὀρθολογία is otherwise unattested in Attic Greek (see LSJ[9], *s.v.*).

places with two terms distributed through them. The model of proportion, as applied to the case of statement, must be simplified accordingly.

The main parallel to which we are calling attention gives rise to the following question. We have emphasized that the proportions into which we analyze assertions that a given statement is true or false put the same objects on both sides of the division between statement and being: does this not collapse the true statement with the fact it states? Readers of Russell's *Problems of Philosophy* (London 1912), Chapter 12, are often vexed by a similar puzzle in his doctrine of false belief, which is in many ways like the doctrine of the *Sophist*. If and only if it is true what Othello believes, i.e., that Desdemona loves Cassio, then there exists such a complex as Desdemona's love for Cassio (or, that Desdemona loves Cassio), and this, though its actual existence is independent of Othello's mind, is composed of the very objects which also go to compose his belief. But how, one wonders, can the objects of the world *be* the very objects in the believer's mind? In reply, one might ask, how can they *fail* to be the very objects concerning which he has belief? It seems a reasonable answer to this question simply to say that it is the same thing that can be believed and can be. More fully, the same relation which is believed to hold among objects, or holds among them in a picture, can also hold among them in reality, and does so just when the belief or picture is true to reality. Similarly, it is the same thing that one states to be the case with certain objects and which is the case when the statement is true, or not the case when it is false.

Finally, a point about the Academy in the mid-fourth century. If we are right in finding a strict parallel between these philosophical and mathematical researches into "not-being in logos" at the Academy, we would have found some confirmation of the familiar Platonic thesis that mathematics prepares the way for philosophy. Nor would it be any surprise if Plato, admiring Theaetetus' work on incommensurability, should have developed his own treatment of false statement so as to run parallel to it, and accordingly had good reason for assigning to this mathematician a central role in the *Sophist*.[15]

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[15]We are indebted to the late Glenn Morrow, and to Ian Mueller, Edward N. Lee, and a reader for *Phoenix* for their criticisms of an earlier draft of this paper.